

Neutrinos, Large Extra Dimensions and Solar Neutrino Puzzle*

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A desirable feature of models with large extra dimensions and a TeV range string scale is the possibility to understand an ultralight sterile neutrino needed for a simultaneous understanding of solar, atmospheric and LSND results as a Kaluza-Klein state of a bulk neutrino. How these oscillation data are understood in quantitative detail is still not clear. We have recently suggested a new way to understand the solar neutrino data in this framework by a combination of vacuum and small angle MSW transition of ν_e to sterile KK modes of the bulk neutrino. This mechanism can be embedded into an economical extension of the standard model in the brane to understand the atmospheric and LSND data. After a brief discussion of how small neutrino masses arise naturally in extra dimensional models, we review this new suggestion.

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I. INTRODUCTION

One of the important predictions of string theories is the existence more than three space dimensions. For a long time, it was believed that these extra dimensions are small and are therefore practically inconsequential as far as low energy physics is concerned. However, recent progress in the understanding of the nonperturbative aspects of string theories have opened up the possibility that some of these extra dimensions could be large [1,2] without contradicting observations. In particular, models, where some of the extra dimensions have sizes as large as a milli-meter and where the string scale is in the few TeV range have attracted a great deal of phenomenological attention in the past two years. The basic assumption of these models, largely inspired by generic observations in string theories is that the space time has a brane-bulk structure, where the brane is the familiar (3+1) dimensional space-time, with the standard model particles and forces residing in it and bulk consists of all space dimensions where gravity and other possible gauge singlet particles live. The main interest in these models is of course due to the fact that the low string scale provides an opportunity to test them using existing collider facilities.

A major challenge to these theories comes from the neutrino sector, the first problem being how one understands the small neutrino masses in a natural manner. The conventional seesaw [3] explanation which is believed to provide the most satisfactory way to understand this, requires that the new physics scale (or the scale of $SU(2)_R \times U(1)_{B-L}$ symmetry) be around 10^{12} GeV or higher and clearly does not work for these models. So one must look for alternative ways. The second problem is that if one considers only the standard model group in the brane, operators such as $L\bar{L}H/M^*$ could be induced by string theory in the low energy effective Lagrangian. For TeV scale strings this would obviously lead to unacceptable neutrino masses.

One mechanism suggested in Ref. [4] is to postulate the existence of one or more gauge singlet neutrinos, ν_B in the bulk which couple to the lepton doublets in the brane. After electroweak symmetry breaking this coupling can lead to neutrino Dirac masses, which are suppressed by the ratio M_*/M_{Pl} . This is sufficient to explain small neutrino masses and owes its origin to the large bulk volume that suppresses the effective Yukawa couplings of the Kaluza-Klein (KK) models of the bulk neutrino to the brane fields. In this class of models, naturalness of small neutrino mass requires that one must assume the existence of a global B-L symmetry in the theory, since that will exclude the undesirable higher dimensional operators from the theory.

An alternative possibility [5] is to consider the brane theory to have an extended gauge symmetry which contains B-L symmetry as a subgroup, which will eliminate the higher dimensional operators. This is perhaps more in the spirit of string theories, which generally do not allow global symmetries. Phenomenological considerations, however, require that the local $B - L$ scale and hence the string scale be of order of 10^{10} GeV or so. The extra dimensions in these models could of course be large.

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Regardless of which path one chooses for understanding small neutrino masses, a very desirable feature of these models is that if the size of extra dimensions is of order of a milli-meter, the KK excitations of the bulk neutrino have masses in the range of 10^{-3} eV, which is in the range needed for a unified understanding of oscillation data [7].

In this paper, we will focus on TeV scale models and discuss their implications. In this class of models, one gets for the Dirac masses of the familiar neutrinos $\nu_{e,\mu,\tau}$

$$m_0 = \frac{h v_{wk} M_*}{M_{Pl}}. \quad (1)$$

The righthanded neutrino in the Dirac mass is a mode of the bulk neutrino ν_B . For $M_* \sim 10$ TeV, this leads to $m_\nu \simeq 10^{-4} h$ eV. It is encouraging that this number is in the right range to be of interest in the discussion of solar neutrino oscillation if the Yukawa coupling h is appropriately chosen.

If one takes these models for the neutrinos seriously, some immediate questions arise: first, while the KK modes of the bulk neutrinos are candidates for sterile neutrinos if the bulk radius is in the range of millimeters, are their small masses natural? Secondly, how well do the models of this type describe the observed oscillation data from solar, atmospheric and LSND experiments? It is clear that they provide a tempting possibility to simultaneously explain all data since the needed sterile neutrino could be one of the KK modes of the bulk neutrino, as already emphasized.

We discuss the answer to the first question in section 2. As far as the second question is concerned, several recent papers have addressed this issue [6,8–11]. In particular, in Ref. [9] it has been shown that while the overall features of the solar and atmospheric data can be accommodated in minimal versions of these models with three bulk neutrinos, it is not possible to simultaneously explain the LSND observation for the $\nu_\mu - \nu_e$ oscillation probability, and one must incorporate new physics in the brane.

On a phenomenological level, a first glance at the values of the parameters of the model such as m_0 from Eq.(1) and $R^{-1} \sim 10^{-3}$ eV suggests that perhaps one should seek a solution of the solar neutrino data in these models using the small angle MSW mechanism [6]. However, present Super-Kamiokande recoil energy distribution seems to disfavor such an interpretation, although any definitive conclusion should perhaps wait till more data accumulates [12]. In any case if the present trend of the data near the higher energy region of the solar neutrino spectrum from Super-Kamiokande persists, it is likely to disfavor the small angle MSW solution and favor a vacuum oscillation. We therefore pursue the possibility of vacuum oscillation between the ν_e and ν_s to solve the solar neutrino puzzle motivated by a model that leads to desired parameters in a natural manner [13].

As is known from many discussions in literature, chlorine results play a pivotal role in deciding on the nature of the oscillation solution to the solar neutrino puzzle. To fit water and the Gallium [19] data in conjunction with the Chlorine data, one must “kill” the ^7Be neutrinos almost completely. In the vacuum solution, it is in general hard to achieve this without simultaneously suppressing the pp neutrinos. Therefore, the general tactic adopted is to suppress the ^7Be neutrinos as much as possible (but not quite to zero) and suppressing the ^8B neutrinos to less than 30% or so. This, however, suppresses the higher energy ^8B contributions below the level observed by Super-Kamiokande collaboration. In the case of oscillation to active neutrinos, this can be fixed by including the neutral current scattering which contributes about 16% of the charged current data. On the other hand, this disfavors the vacuum oscillation to sterile neutrinos which do not have a neutral current component. As we will show later [13], if the sterile neutrino is a bulk mode, its KK modes play a significant role in resolving this problem due to the existence of MSW transitions of ν_e to these modes for values of extra dimension sizes of interest [6].

II. NATURALNESS OF ULTRA LIGHT STERILE NEUTRINOS IN BRANE-BULK MODELS

Before getting to the detailed discussion of the model, we make some general comments about the naturalness of the ultralightness of the bulk neutrino. The bulk neutrino “self mass” terms are constrained by the geometry of the bulk and could therefore under certain circumstances be zero. If that happens, the only mass of the KK states of the ν_B will arise from the kinetic energy terms such as $\bar{\nu}_B \Gamma^I \partial_I \nu_B$, where $I = 5, 6, \dots$ and will be given by n/R where R is the radius of the extra dimensions. In such a situation, an ultralight $\nu_{B,KK}$ arises naturally.

The key to naturalness of the ultralight bulk neutrino is the geometry that forbids both Dirac and Majorana mass terms. Let us give a few examples. In five dimensions, if we impose the Z_2 orbifold symmetry ($y \rightarrow -y$), then it follows that the Dirac mass vanishes. Now if we impose lepton number symmetry in the brane, the Majorana mass vanishes, leaving us with no mass term for the bulk neutrino in 5-dimensions.

Another interesting example is the 10-dimensional bulk, where the bulk neutrino is a **16**-component spinor, which when reduced to 4-dimensions leads to eight 2-component spinors. The interesting point is that for a **16**-dimensional spinor, one cannot write a Dirac or Majorana mass term consistent with 10-dimensional Lorentz invariance. In this case, there is no need for assuming lepton number to get an ultralight sterile neutrino. A similar situation is also expected in six dimensions if we choose the bulk neutrino to be a 4-component complex chiral spinor.

III. BASIC IDEAS OF THE MODEL AND THE MIXING OF BULK MODES

The model we will present will consist of the standard model in the brane (suitably extended to include some additional Higgs fields) and one bulk neutrino. The bulk may be five, six or higher dimensional; we will assume that only one of those extra dimensions is large. The bulk neutrino will be assumed to couple to ν_e ¹. Let us begin our discussion by focusing on the ν_e in the brane and one bulk neutrino. Obviously, the fields that could propagate in the extra dimensions are chosen to be gauge singlets. Let us denote bulk neutrino by $\nu_B(x^\mu, y)$. It has a five dimensional kinetic energy term and a coupling to the brane field $L(x^\mu)$ given by

$$\mathcal{L} = \kappa \bar{L} H \nu_{BR}(x, y = 0) + \int dy \bar{\nu}_{BL}(x, y) \partial_5 \nu_{BR}(x, y) + h.c., \quad (2)$$

where from the five dimensional kinetic energy, we have only kept the 5th component that contributes to the mass terms of the KK modes in the brane; H denotes the Higgs doublet, and $\kappa = h \frac{M^*}{M_{Pl}}$ the suppressed Yukawa coupling. It is worth pointing out that this suppression is independent of the number and radius hierarchy of the extra dimensions, provided that our bulk neutrino propagates in the whole bulk. For simplicity, we will assume that there is only one extra dimension with radius of compactification as large as a millimeter, and the rest with much smaller compactification radii. The smaller dimensions will only contribute to the relationship between the Planck and the string scale, but their KK excitations will be very heavy and decouple from neutrino spectrum. Thus, all the analysis could be done as in five dimensions.

A second point we wish to note is that we will include new physics in the brane that will generate a Majorana mass matrix for the three standard model neutrinos as follows:

$$\mathcal{M} = \begin{pmatrix} \delta_{ee} & \delta_{e\mu} & \delta_{e\tau} \\ \delta_{e\mu} & \delta_{\mu\mu} & m_0 \\ \delta_{e\tau} & m_0 & \delta_{\tau\tau} \end{pmatrix}. \quad (3)$$

The origin of this pattern of brane neutrino masses will be discussed in a latter section. In this section let's discuss the mixing pattern of the bulk neutrinos with the brane ones. For this we will assume that $m_0 \gg \delta_{ij}$; as a result the $\nu_{\mu,\tau}$ in effect do not affect the mixing between the bulk neutrino modes and the ν_e .

To discuss the effect of the bulk modes, note that the first term in Eq. (2) will be responsible for the neutrino mass once the Higgs field develops its vacuum expectation. The induced Dirac mass parameter will be given by $m = \kappa v$, which for $M^* = 1$ TeV is about $h \cdot 10^{-5}$ eV. Obviously this value depends only linearly on the fundamental scale. Larger values for M^* will increase m proportionally. After introducing the expansion of the bulk field in terms of the KK modes, the Dirac mass terms in (2) could be written as

$$(\nu_e \nu_{0B} \nu'_{B,-} \nu'_{B,+}) \begin{pmatrix} \delta_{ee} & m & \sqrt{2}m & 0 \\ m & 0 & 0 & 0 \\ \sqrt{2}m & 0 & 0 & \partial_5 \\ 0 & 0 & \partial_5 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_{0B} \\ \nu'_{B,-} \\ \nu'_{B,+} \end{pmatrix}, \quad (4)$$

where our notation is as follows: ν'_B represents the KK excitations; the off diagonal term $\sqrt{2}m$ is actually an infinite row vector of the form $\sqrt{2}m(1, 1, \dots)$. The operator ∂_5 stands for the diagonal and infinite KK mass matrix whose n -th entry is given by n/R . This notation was introduced in [5] to represent the infinite mass matrix in a compact manner.

Using this short hand notation makes it easier to calculate the exact eigenvalues and the eigenstates of this mass matrix [9]. Simple algebra yields the characteristic equation

$$m_n = \delta_{ee} + \frac{\pi m^2}{\mu_0} \cot\left(\frac{\pi m_n}{\mu_0}\right), \quad (5)$$

where m_n is the mass eigenvalue [4,6]. The equation for eigenstates is

¹In general, the bulk neutrino will couple to all three species of brane neutrinos; however, when the $\nu_{\mu,\tau}$ masses are much bigger than the mass parameters in the ν_e and ν_B sector, one can decouple the $\nu_{\mu,\tau}$ and restrict oneself to only the ν_e, ν_B sector, to analyse the effect of the bulk neutrino, as we do in the paper.

$$\tilde{\nu}_n = \frac{1}{N_n} \left[\nu_e + \frac{m}{m_n} \nu_{0B} + \frac{\sqrt{2}m (m_n \nu'_{B,-} + \partial_5 \nu'_{B,+})}{m_n^2 - \partial_5^2} \right], \quad (6)$$

where the sum over the KK modes in the last term is implicit, and N_n is the normalization factor given by

$$N_n^2 = 1 + m^2 \pi^2 R^2 + \frac{(m_n - \delta_{ee})^2}{m^2}. \quad (7)$$

Using the expression (6), we can write down the weak eigenstate ν_L in terms of the massive modes as

$$\nu_L = \sum_{n=0}^{\infty} \frac{1}{N_n} \tilde{\nu}_{nL}. \quad (8)$$

Thus, the weak eigenstate is actually a coherent superposition of an infinite number of massive modes. Therefore, even for this single flavour case, the time evolution of the mass-eigenstates involves in principle all mass eigenstates and is very different from the simple oscillatory behaviour familiar from the conventional two or three neutrino case. The survival probability depends strongly on the parameter ξ , reflecting the universal coupling of all the KK components of ν_B with ν_L in (2). We will be interested in the small ξ region of the parameter space. In this case it is easy to see that the coupling of the n th mode is given by ξ/n .

Note that in the limit of $\delta_{ee} = 0$, the ν_e and $\nu_{B,0}$ are two, two-component spinors that form a Dirac fermion with mass m . Once we include the effect of $\delta_{ee} \neq 0$, they become Majorana fermions with masses given by: $m_1 \approx -\delta_{ee}/2 + m$ and $m_2 \approx -\delta_{ee}/2 - m$, and they are maximally mixed; i.e., the two mass eigenstates are $\nu_{1,2} \simeq \frac{\nu_e \pm \nu_{B,0}}{\sqrt{2}}$. Thus as the ν_e produced in a weak interaction process evolves, it oscillates to the $\nu_{B,0}$ state with an oscillation length $L \simeq E/(2m\delta_{ee})$. Later on in this paper we will find that for natural values of m, δ_{ee} in this model, L is of order of the Sun-Earth distance so that our model leads to vacuum oscillation of the solar neutrinos.

Furthermore since the ν_e also mixes with the KK modes of the bulk neutrinos with a mass difference square of order 10^{-6} eV^2 , there is MSW resonance transition of ν_e to $\nu_{B,KK}$ modes. In our discussion of the solar neutrino puzzle, we will take into account both these effects.

In our model, we will also have m_0 in the brane neutrino mass matrix which is of order eV so that we can account for LSND results. Atmospheric neutrino oscillation is now purely ν_μ - ν_τ oscillation.

IV. SOLAR NEUTRINO DATA BY A COMBINATION OF VACUUM AND MSW OSCILLATIONS

Let us now discuss how the solar neutrino data is explained within this model. To discuss the MSW effect for the case of bulk neutrinos, we need to include the matter effect in the diagonalization of the infinite dimensional neutrino mass matrix. The eigenvectors and eigenvalues of this matrix can be found in the presence of matter, when the squared mass matrix, M^2 , is replaced with $M^2 + 2EH_1$, where H_1 is $\rho_e = \sqrt{2}G_F(n_e - 0.5n_n)$ when acting on ν_e , and is zero on sterile neutrinos. Define

$$w_k = \frac{E\rho_e}{m_k\delta_{ee}} + \sqrt{1 + \left(\frac{E\rho_e}{m_k\delta_{ee}} \right)^2}; \quad (9)$$

$w_k = 1$ in vacuum. The characteristic equation becomes

$$m_k = w_k\delta_{ee} + \frac{\pi m^2}{\mu_0} \cot \frac{\pi m_k}{\mu_0}, \quad (10)$$

The eigenvectors are as in Eq. 6, except the coefficients of ν_{0B} and $\nu'_{B,-}$ acquire an additional factor $1/w_n$, and the normalization becomes $N_n^2 = 1 + \frac{(1+\frac{1}{w_n^2})}{2} \left(\pi^2 m^2 R^2 + \frac{(m_n - w_n \delta_{ee})^2}{m^2} \right) - \frac{(1-\frac{1}{w_n^2})}{2} \frac{(m_n - w_n \delta_{ee})}{m}$.

In fitting the solar data the simplest way to reconcile the rates for the three classes of experiments is to “kill” the ${}^7\text{Be}$ neutrinos, reduce the ${}^8\text{B}$ neutrinos by half and leave the pp neutrinos alone. To achieve this in the VO case, one may put a node of the survival probability function P_{ee} around 0.86 MeV. However, for an arbitrary node number, the oscillatory behaviour of P_{ee} before and after 0.86 MeV cannot in general satisfy the other two requirements mentioned above. If one uses the first node to “kill ${}^7\text{Be}$ ”, then for ${}^8\text{B}$ neutrino energies the P_{ee} is close to one and not half as would be desirable; on the other hand, if one uses one of the higher nodes (higher Δm^2), then pp neutrinos get reduced. The strategy generally employed is not to “shoot” for a node at the precise ${}^7\text{Be}$ energy but rather somewhat

away so that it reduces ${}^7\text{Be}$ to a value above zero. This requires that one must reduce the ${}^8\text{B}$ neutrinos by much more than 50%, so one can fit Chlorine data. The water data then requires an additional contribution, which, in the case of active VO, is provided by the neutral current cross section amounting to about 16% of the charged current one. Thus in a pure two-neutrino oscillation picture, VO works for oscillation to active neutrinos but does not work for active to sterile oscillation. It is here that the large extra dimensions come to the rescue as we see below, thereby keeping the active to sterile VO fit to solar neutrino data viable.

To see this note that in our model both vacuum oscillations and MSW oscillations are important. This is because the lowest mass pair of neutrinos is split by a very small mass difference, whereas the KK states have to be separated by $< 10^{-2}$ eV because of the limits from gravity experiments. We can then use the first node of P_{ee} to suppress the ${}^7\text{Be}$. Going up in energy toward ${}^8\text{B}$ neutrinos, the survival probability, which in the VO case would have risen to very near one, is suppressed by the small-angle MSW transitions to the different KK excitations of the bulk neutrino. This is the essence of our new way to fit the solar neutrino observations [13].

To carry out the fit, we studied the time evolution of the ν_e state using a program that evolved from one supplied by W. Haxton [14]. The program was updated to use the solar model of BP98 [15] and modified to do all neutrino transport within the sun numerically. For example, no adiabatic approximation was used. Changes were also necessary for oscillations into sterile neutrinos and to generalize beyond the two-neutrino model. Up to 16 neutrinos were allowed, but no more than 14 contribute for the solutions we considered.

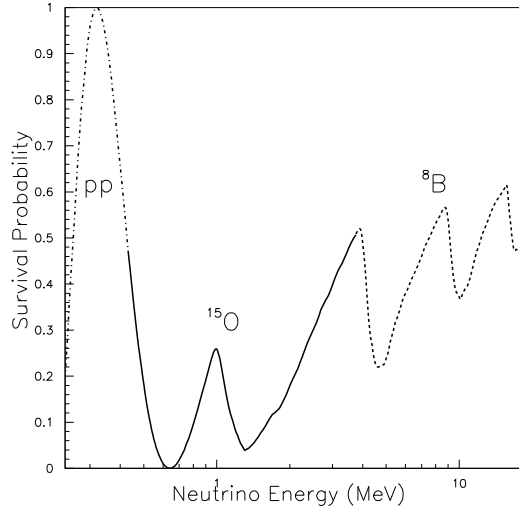


FIG. 1. Energy dependence of the ν_e survival probability when $R = 58\mu\text{m}$, $mR = 0.0093$, $\delta_{ee} = 0.84 \times 10^{-7}$ eV. The dot-dashed part of the curve assumes the radial dependence in the Sun for neutrinos from the pp reaction, the solid part assumes ${}^{15}\text{O}$ radial dependence, and the dashed part assumes ${}^8\text{B}$ radial dependence.

For comparison with experimental results, tables of detector sensitivity for the Chlorine and Gallium experiments were taken from Bahcall's web site [15]. The Super-Kamiokande detector sensitivity was modeled using [16], where the percent resolution in the signal from Cerenkov light, averaged over the detector for various total electron energies, is provided. For details see [13].

Calculations of electron neutrino survival probability, averaged over the response of detectors, were compared with measurements. While theoretical uncertainties in the solar model and detector response were included in the computation of χ^2 as described in Ref. [17], the measurement results given here include only experimental statistical and systematic errors added in quadrature. The Chlorine survival probability, from Homestake [18], is 0.332 ± 0.030 . Gallium results [19] for SAGE, GALLEX and GNO were combined to give a survival probability of 0.579 ± 0.039 . The 5.5 – 20 MeV 1117 day Super-K experimental survival probability [19] is 0.465 ± 0.015 . The best fits were with $R \approx 58\mu\text{m}$, mR around 0.0093, and $\delta_{ee} \sim 0.84 \times 10^{-7}$ eV, corresponding to $\delta m^2 \sim 0.53 \times 10^{-11}$ eV². These parameters give average survival probabilities for Chlorine, Gallium, and water of 0.386, 0.533, and 0.460, respectively. They give a ν_e survival probability whose energy dependence is shown in Fig. 1. For two-neutrino oscillations, the coupling between ν_e and the higher mass neutrino eigenstate is given by $\sin^2 2\theta$, whereas here the coupling between ν_e and the first KK excitation replaces $\sin^2 2\theta$ by $4m^2 R^2 = 0.00035$.

Vacuum oscillations between the lowest two mass eigenstates nearly eliminate electron neutrinos with energies of

.63MeV/(2n+1) for $n = 0, 1, 2, \dots$. Thus Fig. 1 shows nearly zero ν_e survival near 0.63 MeV, partly eliminating the ${}^7\text{Be}$ contribution at 0.862 MeV, and giving a dip at the lowest neutrino energy. Increasing δ_{ee} moves the low energy dip to the right into Gallium's most sensitive pp energy range, making the fit worse. Decreasing δ_{ee} increases Gallium, but hurts the Chlorine fit by moving the higher energy vacuum oscillation dip further to the left of the ${}^7\text{Be}$ peak. MSW resonances start causing the third and fourth eigenstates to be significantly occupied above ~ 0.8 MeV, the fifth and sixth eigenstates above ~ 3.7 MeV, the 7'th and 8'th above ~ 8.6 MeV, and the 9'th and 10'th above ~ 15.2 MeV. Fig. 1 shows dips in survival probability just above these energy thresholds. The typical values of the survival probability within the ${}^8\text{B}$ region (~ 6 to ~ 14 MeV) are quite sensitive to the value of mR . As can be seen from Eq. 7, higher mR increases $1/N \approx m/m_n \approx mR/n$ for various n , and thereby increases ν_e coupling to higher mass eigenstates, strengthens MSW resonances, and lowers ν_e survival probability.

The expected energy dependence of the ν_e survival probability is compared with preliminary Super-K data [19] in Fig. 2. The uncertainties are statistical only. The parameters used in making Fig. 2 were chosen to provide a good fit to the total rates only; they were not adjusted to fit this spectrum. But combining spectrum data with rates using the method described in Ref. [20], with numbers supplied by Super-K [21], gives $\chi^2 = 15.6$ ("probability" 74%) for the spectrum predicted from the fit to total rates. It's incorrect to calculate probability as if there were (number of data points) - (number of parameters) = 17 degrees of freedom, but if we do, χ^2 corresponds to a "probability" of 55%.

One may also seek fits with δ_{ee} constrained to be very small, thereby eliminating vacuum oscillations. The best such fit had $\chi^2 = 5.5$ ("probability" 14%). The same parameters then used with the Super-K spectrum gave $\chi^2 = 18.7$ ("probability" 35%).

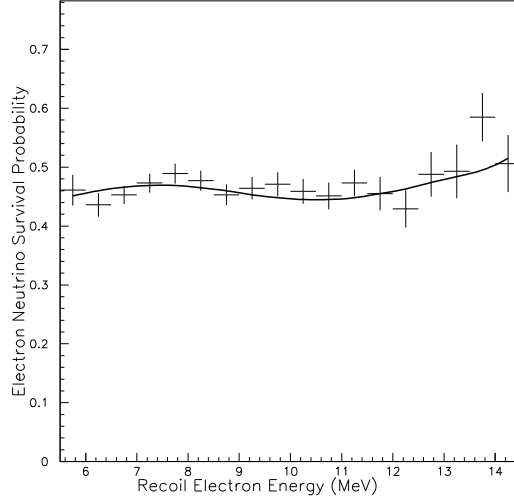


FIG. 2. Super-Kamiokande energy spectrum: measured [19] preliminary results based on 1117 days (error bars) and predicted (curve) for the same parameters as in Fig. 1. The curve is not a fit to these data.

The seasonal effect was computed for a few points on the earth's orbit. If r is the distance between the earth and the sun,

$$\frac{r_0}{r} = 1 + \epsilon \cos(\theta - \theta_0), \quad (11)$$

where r_0 is one astronomical unit, $\epsilon = 0.0167$ is the orbital eccentricity, and $\theta - \theta_0 \approx 2\pi(t - t_0)$, with t in years and $t_0 = \text{January 2, 4h 52m}$. Table I shows very small seasonal variation.

To understand the mass pattern used, consider $L_e + L_\mu - L_\tau$ symmetry for neutrinos, with $L_e = 1$ for ν_B . The allowed mass pattern for $(\nu_e, \nu_\mu, \nu_\tau)$ is given by Eq. 5 with all δ 's (except $\delta_{e\tau, \mu\tau}$) set to zero. The remaining δ entries arise after we turn on the symmetry breaking. For an explicit realization, we augment the standard model by the singlet charged Higgs η^+ , h^{++} which are blind with respect to lepton number and $SU(2)_L$ triplet fields, $\Delta_{e,\mu,\tau}$, with $Y = 2$ which carry two negative units of lepton numbers $L_{e,\mu,\tau}$, respectively. The Lagrangian involving these fields consists of two parts: one \mathcal{L}_0 which is invariant under $(L_e + L_\mu - L_\tau)$ number and contains terms $(\eta L_\mu L_\tau, \eta L_e L_\tau, h^{++} e_R^- \tau_R^-, h^{++} \mu_R^- \tau_R^-)$ and $L_e L_e \Delta_e, L_\mu L_\mu \Delta_\mu, L_\tau L_\tau \Delta_\tau$ and a soft breaking term $\mathcal{L}_1 = h^{++} (\sum_{i=e,\mu,\tau} M_{ii} \Delta_i^2 + M_{0\mu} \Delta_\mu \Delta_\tau + M_{0\tau} \Delta_e \Delta_\tau) + h.c.$

With these couplings, the neutrino Majorana masses arise from two-loop effects (similar to the mechanism of ref. [22]), and we have the $L_e + L_\mu - L_\tau$ violating entries $\delta_{ij} \sim c m_{e_i} m_{e_j}$. Using $\delta_{ee} \sim 10^{-8}$ eV, then $\delta_{\tau\tau} \sim 10^{-2}$, as would be required to understand the atmospheric neutrino data.

Finally, models with bulk sterile neutrinos lead to new contributions to low energy weak processes, which have been addressed in several papers [23]. In the domain of astrophysics, they lead to new contributions to supernova energy loss, as well as to the energy density in the early universe, which can influence the evolution of the universe. Currently these issues are under discussion [24,8,25], and if neutrino oscillation data favor these models, any cosmological constraints must be addressed. Note, however, that this model is less sensitive to these issues than other fits because for VO Δm^2 is an order of magnitude smaller, for MSW $\sin^2 2\theta$ is more than an order of magnitude smaller, and there is only a single KK tower starting from a very small mass [24].

TABLE I. Predicted seasonal variations in ν_e fluxes, excluding the $1/r^2$ variation. The model assumed $\mu_0 = 0.32 \times 10^{-2}$ eV, $m_0 = 0.34 \times 10^{-4}$ eV, and $\delta_{ee} = 0.78 \times 10^{-7}$ eV.

$\theta - \theta_0$ in eqn 11	Chlorine	Gallium	Water
0 (January 2)	0.3787	0.5144	0.4635
$\pm\pi/2$	0.3762	0.5121	0.4633
π (July 4)	0.3747	0.5082	0.4631

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